

CALCULATION OF THE PARAMETERS OF THE PLASMA FLOW
OF PULSED EROSION PLASMA ACCELERATOR

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UDC 533.9

An investigation of pulsed electromagnetic erosion-plasma accelerators is of interest for a wide range of scientific and applied problems (to generate high-enthalpy plasma jets, to obtain intense shock waves, high-power radiation fluxes in the visible, UV, and vacuum parts of the spectrum, etc. (see, for example, [1, 2])). The processes in a pulsed electromagnetic accelerator are extremely complex: The plasma flows are extremely nonuniform, the plasma is nonideal and nonequilibrium, radiation and ionization play an important role, erosion of the construction occurs and there are energy transfer processes from the store, etc. Existing numerical models of electromagnetic accelerators [3-5] are designed mainly to analyze the mechanism of these phenomena, they are extremely complex, and contain different important assumptions which may considerably distort the integral parameters of the plasma flows. On the other hand, to match the accelerator to the energy store, to carry out practical calculations, and to solve a number of other problems not requiring detailed information on the processes occurring in the accelerator, it is more convenient to use relatively simple and fairly reliable semiempirical methods, which relate the characteristics of the equipment to the parameters of the plasma flows. These models are well known for pulsed accelerators operating in the single-bunch generation mode (the "plasma-disk" model proposed by Artsimovich [6]) and for accelerators in which the gas is admitted (the "snow-clearing" model [7]); using them a number of optimization investigations have been made ([8, 9], etc.). However, direct application of these models to erosion-type pulsed electromagnetic accelerators leads to a qualitatively incorrect description of the dynamics of the processes.

In this paper we construct a new electrodynamic model which takes into account the particular features of quasistationary acceleration, erosion plasma formation, and the transfer of energy from the store. It enables the time dependences of the velocity and plasma-flux density to be determined, and enables pulsed electromagnetic accelerators to be matched to energy stores of different kinds (capacitive, inductive, etc.). A numerical analysis of this model enabled us to investigate a number of features of the processes, and to determine methods of optimizing the parameters of equipment with pulsed electromagnetic accelerators.

1. As is well known, the pulsed electromagnetic accelerator is a system of coaxial rail-shaped or plane-parallel electrodes, by means of which a high-current discharge is set up, and the configuration of the magnetic fields of the discharge is such that the electrical-discharge plasma is accelerated due to the action of its own magnetic forces. In erosion-type accelerators, the plasma is formed from the constructional materials, which erode due to the effect of the thermal flux from the discharge zone; a time delay is observed between the output of the mass and the thermal flux (the "inertia" of plasma formation) (see, for example, [7]).

A characteristic feature of an erosion-type pulsed electromagnetic accelerator which makes it similar to stationary high-current accelerators, is the quasistationary mode of plasma acceleration, when the acceleration time of an element of mass $\tau_y \ll T_{1/2}$, where $T_{1/2}$ is the half-period of the discharge. To take this into account, we will represent the plasma flux in the form of a series of individual bunches; we will describe the acceleration of the i -th bunch in the electrodynamic approximation [6]

$$m_i dv_i/dt = 0L'I^2/2, dz_i/dt = v_i \quad (1.1)$$

up to the instant t_i when it leaves the acceleration zone, which is found from the condition

$$t_i - t_{i-1} = \tau_y, \quad (1.2)$$

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 3-9, March-April, 1984. Original article submitted February 24, 1983.

after which the motion of the i -th bunch is assumed to be uniform, and the $(i + 1)$ -th bunch begins to be accelerated, where we assume

$$v_{i+1}(t_i) = 0, z_{i+1}(t_i) = 0, I(t_i + 0) = I(t_i - 0). \quad (1.3)$$

In relations (1.1)-(1.3) we have used the following notation: m_i , v_i , and z_i are the mass, velocity, and coordinate of the i -th bunch, I is the discharge current, L' is the inductance per unit length of the electrodes, and $\theta = 2$ is a factor which takes into account the effect of instantaneous changeover of the current from the i -th to the $(i + 1)$ -th bunch.[†]

The effective resistance of the pulsed electromagnetic accelerator, required to calculate the energy transfer from the store, can be calculated from the relation

$$R_e(t) = \theta L' v_i(t) / (2\eta_*), \quad (1.4)$$

which differs from the corresponding expression used in [6], by the introduction of the factor $\eta_* = 0.6-0.9$ — the ratio of the kinetic energy of the flux to the total energy introduced into the pulsed electromagnetic accelerator (the kinetic efficiency of the accelerator, which takes into account the loss due to ionization, radiation, heating of the electrodes, etc.). The effective resistance is proportional to v_i , which varies from 0 to the velocity of the emitted flux v . It can be shown that the pulsation in R_e produced by this leads only to oscillations of the current and voltage on the store with an amplitude of the order of $\tau_y / T_{1/2} \ll 1$ and frequency $1/\tau_y$, and these oscillations disappear as $\tau_y \rightarrow 0$. In fact, by considering, to be specific, the beginning of the discharge of a capacitive store with ideal current conductors, we have from Kirchhoff's law,

$$dI/dt = (U - R_e I) / (L_p + L' z_i),$$

where U is the voltage on the store, and L_p is the parasitic inductance of the current conductors of the capacitive store. The drop in voltage $R_e I$ across the accelerator does not exceed U , hence $dI/dt > 0$, i.e., the currents increase monotonically over the whole interval τ_y , and $I(t_i) < I(t') < I(t_{i+1})$ when $t_i < t' < t_{i+1}$, so that taking (1.3) into account we obtain that the deviation of the relationship $I = I(t)$ from the curve averaged over the pulsations does not exceed $I(t_{i+1}) - I(t_i) \sim I(t_i) \tau_y / T_{1/2}$ (even when $L_p = 0$, when $t = t_i + 0$ the rate of growth of the current is infinitely large). For the function $U = U(t)$ these oscillations are even less, since only the derivative $dU(t)/dt$ proportional to $I(t)$ oscillates. A consideration of any other period of the discharge and type of store leads to similar conclusions.

The mass of the bunch can be expressed in terms of the flow rate \dot{m} of the plasma-forming material

$$m_i = \int_{t_{i-1}}^{t_i} \dot{m} dt. \quad (1.5)$$

Using Eqs. (1.1)-(1.4) it can be shown that with this definition of m_i when $\tau_y \ll T_{1/2}$, the value of τ_y has no effect on the electrical characteristics of the discharge and the parameters of the outflowing plasma flux, and it merely defines the degree of discreteness, and in the limit as $\tau_y \rightarrow 0$, it models the acceleration of a continuous flux. More correctly, a

reduction of τ_y to zero reduces L_Σ by an amount $\Delta L = (1/\tau_y) \int_{t_{i-1}}^{t_i} L' z_i dt \simeq L' z_*/3$ where $z_* = z_i(t_i)$

is the length of the acceleration zone; as an analysis of the experimental data shows (see, for example [10, 11]) in erosion accelerators $z_* \leq 0.5$ cm, and usually $\Delta L \ll L_\Sigma$.

Since a reduction in τ_y has no effect on the result of the calculation, it is possible to transfer to the limit of continuous flow $\tau_y \rightarrow 0$, and instead of Eqs. (1.1)-(1.5) to derive explicit analytical expressions for the rates of flow and R_e : Integrating these equations

[†]In the model, when each bunch is emitted, the volume occupied by the magnetic field between the bunches containing an energy $L' z_i(t_i) I^2 / 2 = m_i [v_i(t_i)]^2 / 2$ is "cut off." In fact, this energy participates in the acceleration and to take it into account we have introduced the factor θ .

with respect to τ_y , taking into account the fact that as $\tau_y \rightarrow 0$ the functions $I = I(t)$ and $\dot{m} = \dot{m}(t)$ undergo only infinitely small changes, we obtain

$$v(t) = \theta L' [I(t)]^2 / [2\dot{m}(t)], \quad R_e(t) = \theta^2 [L'I(t)]^2 / [8\eta_* \dot{m}(t)] = \theta L' v / (4\eta_*). \quad (1.6)$$

These expressions, obtained using the electrodynamic approximation and taking into account the smallness of the acceleration time of an element of mass, describe the quasistationary mode of electromagnetic acceleration. To close the system to (1.6) we must add equations which describe the transfer of energy from the store, and relations for determining the mass yield.

2. The processes in the shaping circuit are described by the well-known equations corresponding to Hirschhoff's laws. The kinetic efficiency of the equipment, which takes into account the efficiency with which the energy in the store is converted into kinetic energy of the flow, is $\eta_k = \eta_* \eta_S$; here, $\eta_S = W_y / W_\Sigma$ is the efficiency of energy transfer from the store,

$W_y = \int_0^\infty R_e I^2 dt$ is the energy introduced into the pulsed electromagnetic accelerator, and W_Σ

is the initial energy in the store. In the case of a capacitive store the energy losses in the circuit can be taken into account by introducing into the circuit a single resistance R_S ; we then obtain the following expression for the kinetic efficiency of the equipment:†

$$\eta_k = \eta_* \langle R_e \rangle / (\langle R_e \rangle + R_S), \quad \langle R_e \rangle = \int_0^\infty R_e I^2 dt \left/ \int_0^\infty I^2 dt \right. \quad (2.1)$$

In this method of analyzing an erosion-type pulsed electromagnetic accelerator, the flow of mass, according to the above-mentioned features of the erosion mechanism of plasma formation, can be found from the equation

$$\dot{m} = q/r - T_* \dot{m}/dt, \quad T_* = \text{const}, \quad (2.2)$$

where q is the thermal flux, r is the specific heat of formation of the plasma, the last term takes into account the effect of the inertia of the mass emission, and T_* is the inertia parameter. Since the plasma heating is due to the flow of discharge currents through it, we will assume that a functional relationship $q = q(I)$ exists. Its specific form is obtained from experimental data [12] giving relationships between the mass evaporated during the discharge as a function of W_Σ . For this, the initial relationship [$m_\Sigma = m_\Sigma(W_\Sigma)$] and the required relationship [$q = q(I)$] were written in the form of power series, and the function $m_\Sigma = m_\Sigma(W_\Sigma)$ was represented in terms of the integral of the desired function. It was assumed that the level of the currents when the capacitor battery discharges is proportional to the initial voltage, and the time dependence of the current differs only slightly from the corresponding curve for an oscillatory circuit (these conditions are usually satisfied with a sufficient degree of accuracy). Using the identical equation of the two representations of the initial function, we obtain the following relationship between the expansion coefficients of the two functions considered and the initial relationship:

$$q/r = m_* \langle A \rangle (0.25 |I| + 0.64 |I^2| + 0.81 |I^3|).$$

Here q/r is expressed in $\text{mg}/\mu\text{sec}$, I is expressed in MA, $m_* = \text{const}$ is the mass-yield coefficient characterizing the specific erosion conditions (the thermal properties of the eroding construction elements, the form of the electrodes, etc.), and $\langle A \rangle$ is the mean atomic weight of the plasma ions.

Equations (2.2) and (2.3) enable the flow rate of the plasma-forming mass to be determined.

3. The method of analyzing an erosion-type pulsed electromagnetic accelerator contains experimental parameters η_* , m_* , and T_* ; it is difficult to determine these parameters directly by experiment with sufficient accuracy. It is more convenient to obtain their values by comparing the experimental relationship $v = v(t)$ and $I = I(t)$ with calculations. It is important that these parameters should vary only slightly when the discharge conditions are

†If we assume that all the mass is accelerated to $v(t) = v_\alpha = \text{const}$, and $\theta = \eta_* = 1$, we obtain from Eqs. (1.6) and (2.1) $\eta_k = 1/[1 + 4R_S/(L'v_\alpha)]$, which is the expression derived in [7] for the kinetic efficiency for the "plasma-disk" model. Hence, for certain simplifying assumptions, the proposed model can give time-integral characteristics which agree with the results of an analysis of the "plasma-disk" model.

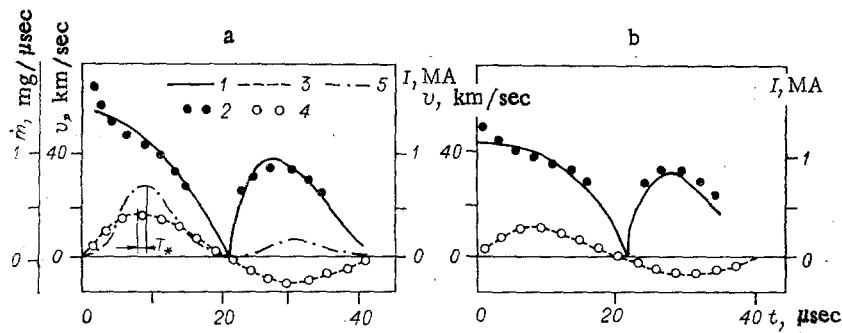


Fig. 1

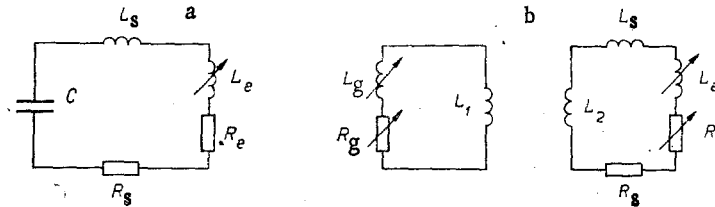


Fig. 2

changed, and they can be used when analyzing other modes of operation and other equipment. Thus, in Fig. 1 we show a comparison of the theoretical and experimental [1] relations for the rate of emission of the flux (curves 1 and 2), and the discharge current (curves 3 and 4), and also a theoretical curve 5 for the flow rate of the plasma (the experimental data [11] on $v = v(t)$ obtained for $z = 150$ mm from the cathode section are reduced to $z = 0$). The discharge modes are as follows: a) $W_{\Sigma} = 9.4$ kJ, b) $W_{\Sigma} = 3.4$ kJ, $C = 750$ μ F, $L_s = 50$ nH, $L' = 1.75$ mH/cm, and the working material is PTFE.

For both modes of operation, which differ in energy input by a factor of approximately 3, we used the same set of values of the parameters η_* , T_* , and m_* . The equivalent electric circuit of the discharge is shown in Fig. 2a. The theoretical and experimental values of the velocity are practically identical during the time of maximum energy dissipation, and of most intense plasma formation. On the basis of a comparison of the calculations and experimental data we can conclude that the model considered describes the dynamics of the plasma flow quite well when operating an erosion-type pulsed electromagnetic accelerator with capacitive energy storage.

Using this model of a pulsed electromagnetic accelerator we analyze the operation of an accelerator from an explosive magnetocumulative generator using a matching transformer (Fig. 2b). The inductance of the explosive magnetocumulative generator was specified in the form

$$L_g(t) = L_g^0(1 - t/t^0),$$

where the parameters L_g^0 , t^0 , R_g , etc., corresponded to the equipment described in [13]. We used the same values of the parameters m_* , η_* , and T_* as when analyzing the operation of a pulsed electromagnetic accelerator with capacitive stores. A comparison of the results of the calculations and the experimental data [13] showed that the mean error in determining the values of the current and voltage in the pulsed electromagnetic accelerator does not exceed 15% (Fig. 3), i.e., the proposed model describes with a sufficient degree of accuracy the processes of energy transfer from the explosive magnetocumulative generator to an erosion-type pulsed electromagnetic accelerator. In Fig. 3, curves 1 are calculated and curves 2 and 3 are experimental.

4. Numerical analysis of the model reveals the complex and self-consistent nature of the interaction between the three processes considered, namely, quasistationary acceleration, erosion plasma-formation, and energy transfer from the store. Thus, an increase in the discharge current increases the accelerating forces and intensifies the plasma-formation processes, and the first leads to an increase in the rate of flow, while the second limits this effect; an increase in the velocities increases the effective resistance of the pulsed electromagnetic accelerator, which has the opposite effect on the discharge current. One effect of this is a fairly weak dependence of the mean mass flow rate $\langle v \rangle$ on the mean discharge

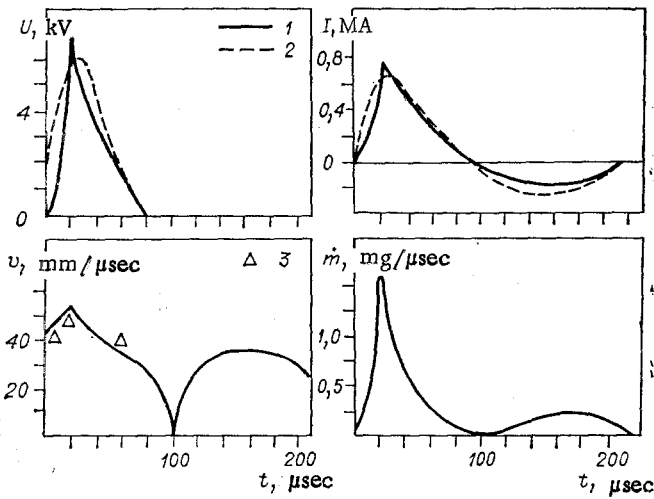


Fig. 3

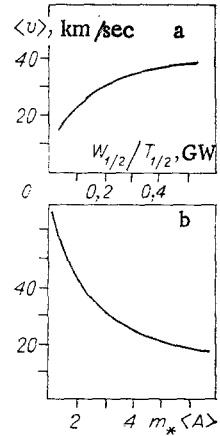


Fig. 4

power $W_{1/2}/T_{1/2}$ (Fig. 4a, where $W_{1/2}$ is the energy supplied in a time $T_{1/2}$), which is also observed experimentally (see, for example, [11]).

A reduction in the erosion plasma-formation intensity coefficient $m_* \langle A \rangle$ leads to an increase in the velocity and, consequently in R_e , which reduces the discharge current and gives rise to an additional reduction in the mass yield (in this case because of the increase in $\langle R_e \rangle$ the efficiency of energy transfer from the store increases). A consequence of this is the considerable dependence of the mean-mass velocity on the plasma-formation intensity coefficient (Fig. 4b). Hence, a considerable increase in the mean mass velocity can be obtained by using weakly eroding electrodes and high-melting point dielectric.

When the current increases, due to the inertia of the plasma formation, the accelerated masses increase more rapidly than the accelerating forces, and hence when the current increases the rate of emission of the plasma is higher than when the current is falling. This effect begins to play an important role for values of the inertia parameter $T_* \geq 0.1T_{1/2}$, when the time disagreement between the extrema on the current and mass-yield curves, which is equal to $T_* \geq 1 \mu\text{sec}$, is already slightly noticeable (Fig. 1). Thus, an increase in the instantaneous velocity of plasma flow can be obtained by reducing the half-period of the discharge (in this case, the velocity spread of the mass increases).

When $T_{1/2} \gg 10T_*$, the inertia of plasma formation ceases to play any important role, and the velocity has maxima which coincide in time with the current extrema. In this case, $\dot{m}(I) = q(I)/r$, and the determination of the parameters of the plasma flow reduces to calcu-

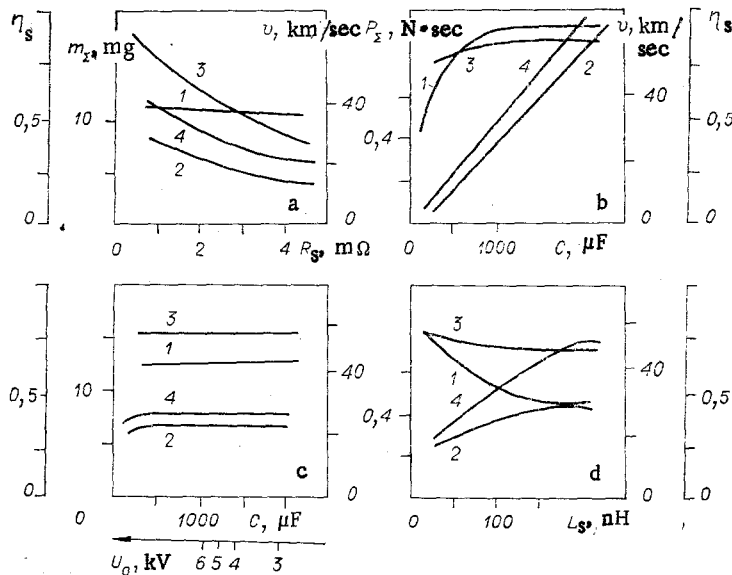


Fig. 5

lating the energy transfer from the store to a nonlinear load $R_e = R_e(I)$.

5. The model described was used to analyze the effect on the parameters of the plasma flow of the characteristics of the capacitive energy store, most often employed for supplying pulsed electromagnetic accelerators. The results of calculations of the mean mass velocity, the total momentum, the kinetic efficiency, and the total mass are shown in Fig. 5 (curves 1-4, respectively). As calculations showed, the mean effective resistance of the accelerator $\langle R_e \rangle$ is 3-8 m Ω over a wide range of variation of the parameters, i.e., for a circuit resistance $R_s > 1$ m Ω the losses when transmitting energy from the store considerably reduce the overall efficiency of the equipment (Fig. 5a).

For a constant initial voltage, as the capacitance of the battery increases, the mean mass velocity increases only slightly; a linear increase in the total momentum can only be achieved by increasing the total accelerated mass (Fig. 5b). A change in the capacitance and the initial voltage of the store for a constant stored energy has only a slight effect on the integral parameters of the discharge (Fig. 5c).

An increase in the inductance of the discharge circuit leads to an increase in the duration of the half-periods of the discharge and their number and a corresponding reduction in their level of the currents, which gives rise to some increase in the overall mass of the plasma and a small reduction in the velocity, whereas the total momentum and efficiency of the equipment change only slightly (Fig. 5d).

As is well known, another conclusion follows from the "plasma-disk" model, namely the considerable effect that the inductance of the store has on the acceleration efficiency. In this model it is assumed that the discharge ceases after the first plasma bunch emerges from the accelerator, when there is still energy in the store; the fraction of this energy increases as the initial inductance of the discharge circuit increases. However, as experimental data show [10, 11] (see Fig. 1), in the case of the quasistationary mode of operation of the pulsed electromagnetic accelerator disconnection of the discharge circuit does not occur, obviously, due to the constant supply of plasma-forming vapor to the interelectrode gap, and acceleration of the plasma continues until the store is completely discharged; this is taken into account in the proposed model. Hence, the inductance of the discharge circuit affects the efficiency of the quasistationary mode of acceleration much less than the efficiency of the single-bunch generation mode.

Hence, the proposed model of the erosion-plasma pulsed electromagnetic accelerator, based on the electrodynamic approximation, unlike the classical "plasma-disk" model, describes the quasistationary mode of acceleration which, together with a consideration of the particular features of the erosion nature of plasma formation, enables a number of features of the change in certain important integral characteristics of equipment with the accelerators to be explained, and enables fairly reliable multiparametric optimization of these systems to be achieved.

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STATIONARY DISCHARGE ACCOMPANYING EMERGENCE OF THE MAGNETIC FLUX
THROUGH THE SURFACE OF AN INSULATOR

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UDC 537.52

It is shown in [1, 2] that when a magnetic flux flows through the surface of an insulator, a stationary surface discharge, which limits the velocity of outflow of the magnetic force lines, can appear. A theory of such a discharge, using a number of simplifying assumptions, in particular the assumption of total single-ionization of the vapor of the insulator flowing out of the discharge, which is valid for not very strong magnetic fields $H \sim 10^4$ Oe, is developed in [2]. In this paper we examine the more general case of arbitrary multiple ionization, which is important for stronger magnetic fields, in particular fields used in experiments on magnetic acceleration of shells (see, for example, [3]).

In the problem under examination the mutually perpendicular magnetic H and electric E fields are parallel to the surface of the insulator, which we assume is flat. The self-sustaining surface discharge along the vapor of the insulator is realized due to the fact that the outflow of plasma driven by the ponderomotive force from the surface is compensated by vaporization of new sections of the insulator by the thermal radiation from the plasma being carried away. The ionized vapor entering into the discharge zone continues to be heated by Joule heat and is accelerated until the plasma velocity reaches the velocity v_1 of the outflow of magnetic force lines and the electric field in the comoving coordinate system vanishes.

Under typical experimental conditions, the thickness of the discharge x_H is much smaller than the dimensions L of the region in which the vapor of the insulator moves (for $H \sim 10^4$ Oe, x_H is of the order of 0.1 cm and decreases with increasing H). For this reason, the time for restructuring of the vaporization regime is much shorter than the characteristic time for the change in the magnetic fields or other quantities that affect the current layer, and the discharge may be assumed to be stationary. In solving the complete magnetohydrodynamic problem, which describes the action of one or another experimental setup, in which such a discharge occurs, the discharge zone can be replaced by an infinitely narrow jump in all MHD quantities. The purpose of this work is to obtain the conditions on this jump; for this it is necessary to find the dependence of the velocity v_1 of outflow of the plasma as well as its density and temperature on the magnitudes of the magnetic fields in the unvaporized insulator H_0 and at the outlet from the current layer H_1 .

A significant factor is that, in order to obtain the dependences indicated, strictly speaking, it is not sufficient to know only the integral laws of conservation, relating the quantities at the inlet and outlet of the discharge zone; it is also necessary to solve the problem of the structure of this zone. In [2] it was possible to circumvent the solution of this more complicated problem only by making an approximation in which the temperature of